Local Church-Rosser, Parallelism and Concurrency

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Motivation

- Can sequentially applied transformations be unified?
  - Yes, if independent, by parallel transformations.
  - Yes, if dependent, by concurrent transformations.
- Can concurrently applied transformations be unified?
  - Yes, if independent, by parallel transformations.
  - Perhaps, if dependent (by local confluence $\rightarrow$ talk on Tuesday).
- Applications of this part of the theory include
  - resolution of concurrent refactoring or editing operations,
  - functional behaviour of model transformations (together with termination $\rightarrow$ talk on Wednesday), and
  - existence and computation of normal forms (again with termination).

Abstract Transformation Systems

- An Abstract Transformation System Framework consists of
  - a class *Objs* of objects,
  - a class *Prods* of productions, and
  - a transformation relation $\Longrightarrow_p \subseteq \mathit{Objs} \times \mathit{Objs}$ for all $p \in \mathit{Prods}$.
- An Abstract Transformation System
  - is a set $P \subseteq \mathit{Prods}$ of productions and
  - induces a transformation relation

$$\Longrightarrow_P \quad := \quad \bigcup_{p \in P} \Longrightarrow_p \quad .$$

- For a transformation relation $\Longrightarrow_P$
  - the inverse is denoted by $\Longleftarrow_P$,
  - the symmetric closure is denoted by $\Longleftrightarrow_P$,
  - the transitive closure is denoted by $\overset{+}{\Longrightarrow}_P$, and
  - the reflexive and transitive closure is denoted by $\overset{*}{\Longrightarrow}_P$.

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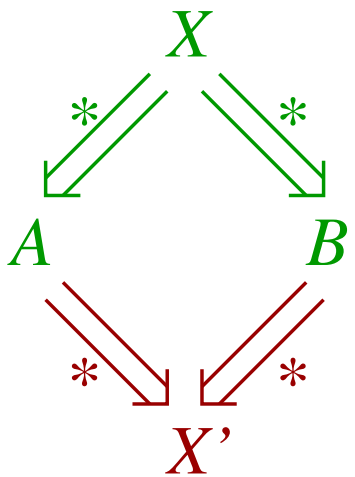
Normal Forms and Termination

- N is normal form of A iff $A \overset{*}{\Longrightarrow}_P N$ and not exists X with $N \Longrightarrow_P X$.
- $\Longrightarrow_P$ terminates iff there is no infinite sequence $(p_i)_{i \in \mathbb{N}}$ of productions $p_i \in P$ with $A_i \Longrightarrow_{p_i} A_{i+1}$.
  ($\rightarrow$ Talk on Wednesday)
- Termination implies existence of normal forms for all objects.
- Existence of normal forms does not imply termination.

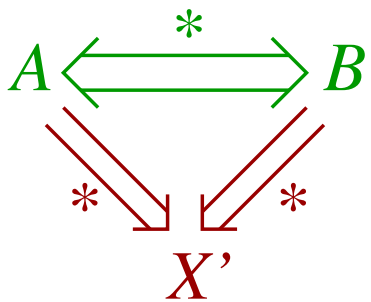
Confluence

- $P \subseteq \mathit{Prods}$ is confluent iff
  for all $X \overset{*}{\Longrightarrow}_P A$ and $X \overset{*}{\Longrightarrow}_P B$
  there exists X' with $A \overset{*}{\Longrightarrow}_P X'$ and $B \overset{*}{\Longrightarrow}_P X'$.
- Confluence implies uniqueness of normal forms for all objects.
- Uniqueness of normal forms does not imply confluence.



Church-Rosser Property

- $P \subseteq \mathit{Prods}$ is Church-Rosser iff
  for all $A \overset{*}{\Longleftrightarrow}_P B$
  there exists X' with $A \overset{*}{\Longrightarrow}_P X'$ and $B \overset{*}{\Longrightarrow}_P X'$.
- Church-Rosser is equivalent to confluence.

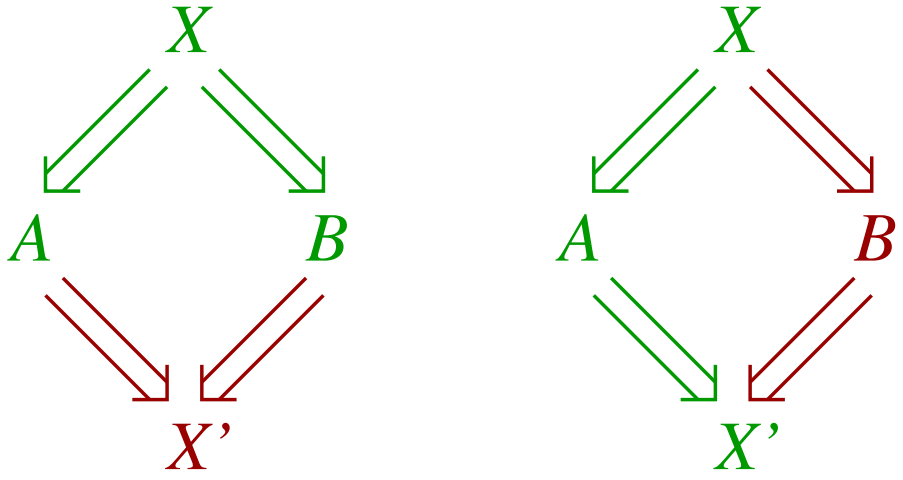


Convergence and Local Confluence

- $P \subseteq \mathit{Prods}$ is convergent iff P is terminating and confluent.
- Convergence implies existence of unique normal forms for all objects.
- Existence of unique normal forms implies confluence, but not termination.
- $P \subseteq \mathit{Prods}$ is locally confluent iff
  for all $X \Longrightarrow_P A$ and $X \Longrightarrow_P B$
  there exists X' with $A \overset{*}{\Longrightarrow}_P X'$ and $B \overset{*}{\Longrightarrow}_P X'$.
  ($\rightarrow$ Talk on Tuesday)
- Confluence implies local confluence.
- Local confluence and termination imply confluence (and convergence).

Local Church-Rosser

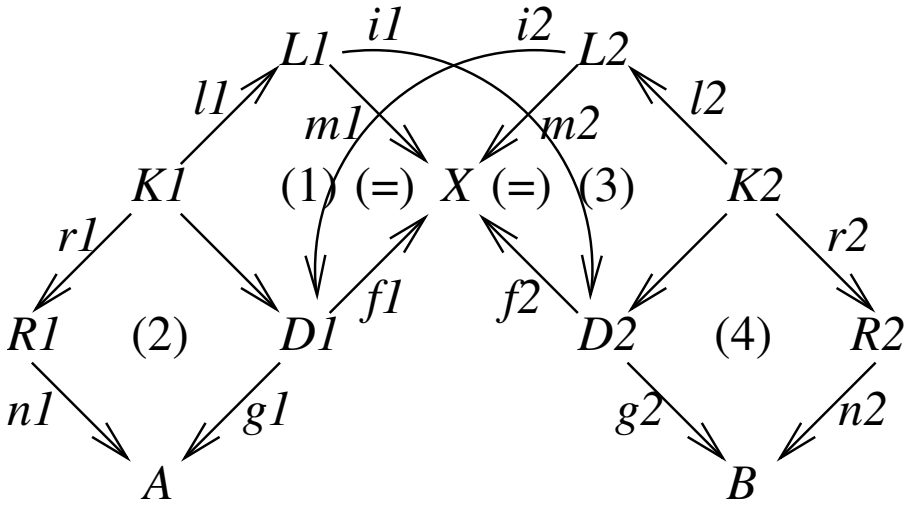
- Local Church-Rosser Theorem:
  1. For all parallely independent $X \Longrightarrow_{p_1} A$ and $X \Longrightarrow_{p_2} B$ there exists X' with $A \Longrightarrow_{p_2} X'$ and $B \Longrightarrow_{p_1} X'$, such that $X \Longrightarrow_{p_1} A$ and $A \Longrightarrow_{p_2} X'$ as well as $X \Longrightarrow_{p_2} B$ and $B \Longrightarrow_{p_1} X'$ are sequentially independent.
  2. For all sequentially independent $X \Longrightarrow_{p_1} A$ and $B \Longrightarrow_{p_2} X'$ there exists B with $X \Longrightarrow_{p_2} B$ and $B \Longrightarrow_{p_1} X'$, such that $X \Longrightarrow_{p_1} A$ and $X \Longrightarrow_{p_2} B$ are parallely independent.



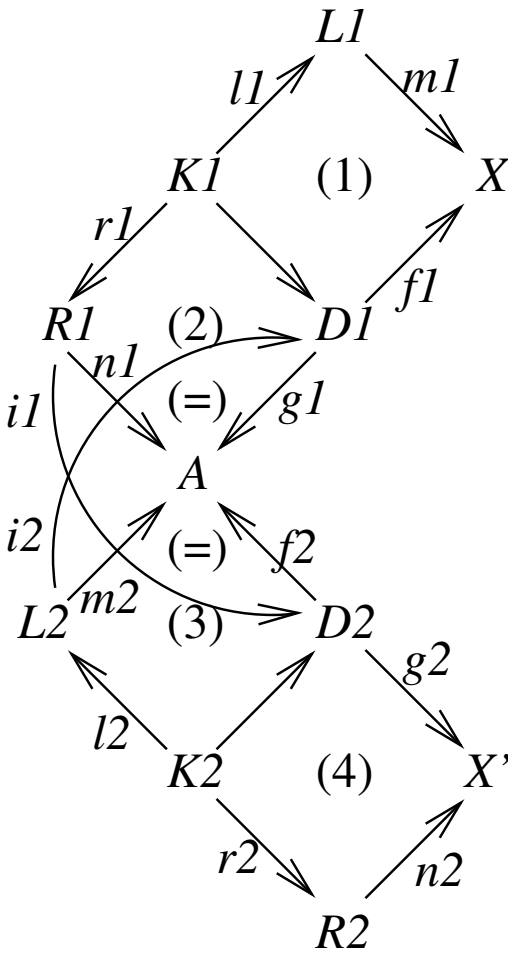
Application of Local Church-Rosser Theorem

- Local confluence is implied by
  - local Church-Rosser and
  - confluence of parallely dependent transformations.
- Confluence of parallely dependent transformations is implied by
  - resolution of critical pairs.
- For finite transformation systems the set of critical pairs is finite.
- $\Rightarrow$ Finite method for proving local confluence

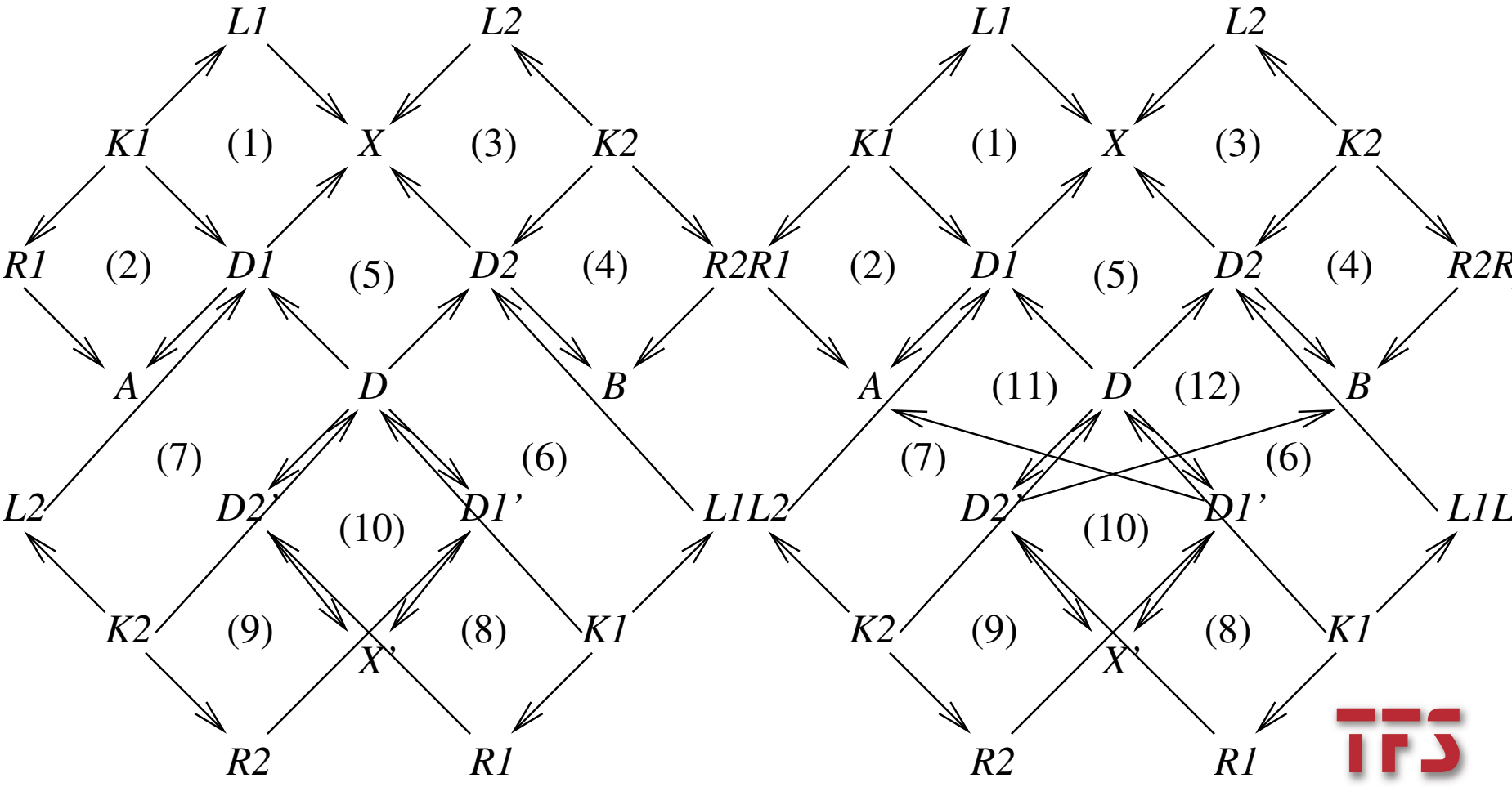
Parallel Independence for AHLR Systems



Sequential Independence for AHLR Systems



Local Church-Rosser for AHLR Systems



Summary

- Parallel Transformations as Unification of Parallely or Sequentially Independent Transformations
- Concurrent Transformations as Unification of Sequentially Dependent Transformations
- Criteria for Parallel Dependent Transformations in Talk on Local Confluence (Tuesday)
- Criteria for Functional Behaviour in Talk on Termination (Wednesday)