

Motivation

Local Church-Rosser, Parallelism and Concurrency

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- Can sequentially applied transformations be unified?
 - Yes, if independent, by parallel transformations.
 - Yes, if dependent, by concurrent transformations.
- Can concurrently applied transformations be unified?
 - Yes, if independent, by parallel transformations.
 - Perhaps, if dependent (by local confluence \rightarrow talk on Tuesday).
- Applications of this part of the theory include
 - resolution of concurrent refactoring or editing operations,
 - functional behaviour of model transformations (together with termination \rightarrow talk on Wednesday), and
 - existence and computation of normal forms (again with termination).

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Abstract Transformation Systems

- An Abstract Transformation System Framework consists of
 - a class *Objs* of objects,
 - a class *Prods* of productions, and
 - a transformation relation $\Rightarrow_P \subseteq \text{Objs} \times \text{Objs}$ for all $P \in \text{Prods}$.
- An Abstract Transformation System
 - is a set $P \subseteq \text{Prods}$ of productions and
 - induces a transformation relation

$$\Rightarrow_P := \bigcup_{P \in P} \Rightarrow_P .$$

- For a transformation relation \Rightarrow_P
 - the inverse is denoted by \Leftarrow_P ,
 - the symmetric closure is denoted by \Leftrightarrow_P ,
 - the transitive closure is denoted by $\overset{+}{\Rightarrow}_P$, and
 - the reflexive and transitive closure is denoted by $\overset{*}{\Rightarrow}_P$.

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Normal Forms and Termination

- N is normal form of A iff $A \overset{*}{\Rightarrow}_P N$ and not exists X with $N \Rightarrow_P X$.
- \Rightarrow_P terminates iff there is no infinite sequence $(p_i)_{i \in \mathbb{N}}$ of productions $p_i \in P$ with $A_i \Rightarrow_P A_{i+1}$. (\rightarrow Talk on Wednesday)
- Termination implies existence of normal forms for all objects.
- Existence of normal forms does not imply termination.

Confluence

- $P \subseteq Prods$ is confluent iff for all $X \xrightarrow{*}_P A$ and $X \xrightarrow{*}_P B$ there exists X' with $A \xrightarrow{*}_P X'$ and $B \xrightarrow{*}_P X'$.
- Confluence implies uniqueness of normal forms for all objects.
- Uniqueness of normal forms does not imply confluence.



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Church-Rosser Property

- $P \subseteq Prods$ is Church-Rosser iff for all $A \xleftrightarrow{*}_P B$ there exists X' with $A \xrightarrow{*}_P X'$ and $B \xrightarrow{*}_P X'$.
- Church-Rosser is equivalent to confluence.



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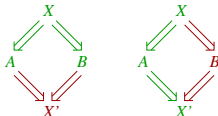
Convergence and Local Confluence

- $P \subseteq Prods$ is convergent iff P is terminating and confluent.
- Convergence implies existence of unique normal forms for all objects.
- Existence of unique normal forms implies confluence, but not termination.
- $P \subseteq Prods$ is locally confluent iff for all $X \xRightarrow{P} A$ and $X \xRightarrow{P} B$ there exists X' with $A \xrightarrow{*}_P X'$ and $B \xrightarrow{*}_P X'$. (→ Talk on Tuesday)
- Confluence implies local confluence.
- Local confluence and termination imply confluence (and convergence).

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Local Church-Rosser

- Local Church-Rosser Theorem:
 - For all parallelly independent $X \xRightarrow{P_1} A$ and $X \xRightarrow{P_2} B$ there exists X' with $A \xRightarrow{P_2} X'$ and $B \xRightarrow{P_1} X'$, such that $X \xRightarrow{P_1} A$ and $A \xRightarrow{P_2} X'$ as well as $X \xRightarrow{P_2} B$ and $B \xRightarrow{P_1} X'$ are sequentially independent.
 - For all sequentially independent $X \xRightarrow{P_1} A$ and $B \xRightarrow{P_2} X'$ there exists B with $X \xRightarrow{P_2} B$ and $B \xRightarrow{P_1} X'$, such that $X \xRightarrow{P_1} A$ and $X \xRightarrow{P_2} B$ are parallelly independent.

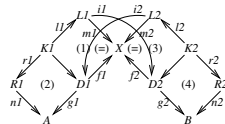


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Application of Local Church-Rosser Theorem

Parallel Independence for AHLR Systems

- Local confluence is implied by
 - local Church-Rosser and
 - confluence of parallelly dependent transformations.
- Confluence of parallelly dependent transformations is implied by
 - resolution of critical pairs.
- For finite transformation systems the set of critical pairs is finite.
- ⇒ Finite method for proving local confluence

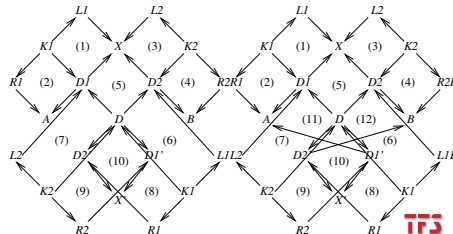
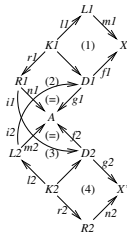


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Sequential Independence for AHLR Systems

Local Church-Rosser for AHLR Systems



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Summary

- Parallel Transformations as Unification of Parallely or Sequentially Independent Transformations
- Concurrent Transformations as Unification of Sequentially Dependent Transformations
- Criteria for Parallel Dependent Transformations in Talk on Local Confluence (Tuesday)
- Criteria for Functional Behaviour in Talk on Termination (Wednesday)

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